Economic Evaluation of Structures of Different Shapes and Materials with Processing Considerations

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Introduction

A computer program was developed to help students perform economic evaluations of structures with different shapes, materials, and processing factors in a more efficient manner. To develop the various programs necessary to evaluate different shapes was very time consuming, and the program is used to evaluate approximately 80 percent of the shapes assigned. The students must develop programs for two or three additional shapes. This permitted the project teams to be reduced in size which increased the ability of the teams to meet and increased the amount of shapes that could be investigated. The purpose of this paper is to illustrate the logic of the program with its benefits over the current approach (developing separate spreadsheets), to present an example of the program input and results, and to describe the typical students using the program.

The student teams previously have used spreadsheets to solve the problem, but when they investigated complex shapes, such as the hollow box, the expressions for the moment of inertia lead to complex expressions requiring search algorithms. The teams often made serious derivation errors and programming errors and obtain poor results with the different shapes. Previously the teams would assign one person for a particular shape, as they were requested to evaluate at least five different shapes and would have five different programs.

The use of a computer model reduced the programming errors made and all teams can easily obtain the correct results for most of the shapes. The student teams develop programs for only the additional two or three shapes. This permits smaller teams and help the students participate at a higher level. The students evaluate their team members for their participation level and the evaluation is used to adjust the project grade for an individual student. The use of the computer has permitted the team size to be reduced to 2-3 students in the large classes instead of the 4-7 students that occurred in the past.

The basic manufacturing processes course, which is required by the Industrial Engineering program, the Mechanical Engineering program, and the Aerospace Engineering and Mechanical Engineering dual major program, is a 2 credit course and is typically taken in the
junior year. A design project in the course represents 20-25 percent of the grade and is completed in the first half of the course to avoid conflict with semester projects in other courses. Since different majors are involved, the project teams must include at least one person from each discipline to meet the ABET requirements of an interdisciplinary team project whenever possible. Thirty-two teams of three students and two teams of two students were initially formed from the class of 100 students. After five weeks of the project, only two students have dropped the course and one of those was because of health reasons.

The course prerequisites require that students have previously completed a course with an emphasis on materials and on the basic mechanics of materials involving stress, strain, and deflection. Most of the students are also taking the one-credit manufacturing processes laboratory course which has experiments in traditional machining, metal casting, welding and CAD-CNC machining. The industrial engineering students also have had a previous course with cost estimating and the mechanical engineering students have had additional mechanics of materials instruction.

The primary purpose of the design project is to integrate the use of material properties, mechanical properties, and material costs into a single project. “Traditionally the attitude of designers has been we design it, you build it” which then results into very high cost of the designed products. The primary material property considered is density and other material properties may be considered depending upon the project. The design typically involves two constraints such as load and deflection, but the binding constraint is usually dependent upon the material and structure selected. The cost focus is primarily on the material cost, but expressions for the processing costs are included as more complex shapes will have higher processing costs and costs for excess weight penalty are also included. The project requires the student to evaluate different cross-sections, such as a solid square, solid rectangle, I-beam, hollow square, hollow rectangle, and sometimes solid circular and hollow circular structures. The students are to select the best and 3 next-best materials and shapes on a cost basis, weight basis, and volume basis and then give a final recommendation as to the material and shape they would utilize for the design. Two of the goals of the project are to expose the mechanical engineering students to the importance of cost considerations in design and to expose the industrial engineering students to the importance of design requirements, processing considerations, mechanical property relationships and material properties upon product cost.

Current Approach and Illustrative Problem

The current solution approach is presented in Introduction to Manufacturing Processes and Materials\textsuperscript{2} published by Marcel Dekker in 1999. The illustrative problem is a beam with single point center loading and two end supports. The two constraints are that it must support a load of 200 kN with a maximum deflection of 1mm. The length of the beam is specified at 25 cm and the student team is to select the best material at the lowest cost. (The student team can also select the best material at the lowest weight or the smallest volume). The student team usually has a selection of 10 or more materials as given in Estimating and Costing for the Metal Manufacturing Industries\textsuperscript{3} published by Marcel Dekker in 1992, but for illustrative purposes, only two materials will be considered, carbon steel and aluminum.
The volume can be expressed as the product of the length and the cross-sectional area, and the cost relationship can be written as:

\[ C_u = C_r \times \rho \times A \times L \quad (1) \]

Where:
- \( A \) = cross-sectional area, \( \text{in}^2 (\text{m}^2) \)
- \( L \) = length, \( \text{in}, (\text{m}) \)
- \( C_u \) = unit cost, \$/unit
- \( C_r \) = relative material cost, \$/weight, \$/lb (\$/kg)
- \( \rho \) = density, \( \text{lb/in}^3 (\text{kg/m}^3) \)

The key is to determine the cross-sectional area or thickness (as the width may be specified) and this is determined by the particular constraints. The shape must be specified and for the example problem a square shape will be considered and the thickness will be determined. The first constraint is the load, which is a single point center load, and for a solid bar in bending the relationship is:

\[ \sigma = \frac{M c}{I} \quad (2) \]

Where:
- \( \sigma \) = tensile strength of material, kpsi (MPa)
- \( M \) = moment = \( PL/4 \)
- \( c \) = centroid (in, m) = \( T/2 \)
- \( I \) = moment of inertia = \( (1/12) WT^3 \)

For this particular problem, one has:

- \( P = \text{load, lb (kN)} = 200 \text{ kN} \)
- \( W = \text{Width} \)
- \( T = \text{Thickness = width for square section (T=W)} \)

Equation 2 can be rewritten and solved for \( T \) for the square section as:

\[ T = \left( \frac{3}{2} \frac{PL}{\sigma} \right)^{1/3} \quad (3) \]

Where \( P, L, \) and \( \sigma \) are known values.

The second constraint is the deflection constraint, that is, for single point center loading:

\[ \delta = \frac{PL^3}{48EI} \quad (4) \]

Where:
- \( \delta \) = deflection limit, in or m
- \( E \) = modulus of elasticity, Mpsi or GPa
- \( P, L, \) and \( I \) are as previously defined.
Equation 4 can be rewritten and solved for $T$ for the square section as:

$$T = \left( \frac{PL^3}{4E\delta} \right)^{\frac{1}{4}} \quad (5)$$

Where $P$, $L$, $\delta$, and $E$ are known values.

There are two expressions for $T$, and the value which must be used is the maximum of the two values as that is the value which will assure that both constraints will be satisfied. Thus the expression for $T$ is:

$$T = \max \left\{ \left[ \frac{3}{2} \frac{PL}{\sigma} \right]^{\frac{1}{3}} , \left[ \frac{PL^3}{4E\delta} \right]^{\frac{1}{4}} \right\} \quad (6)$$

Using the values of $P(200 \text{ kN})$, $L(0.25\text{ m})$, and $\delta(0.001\text{ m})$, the expression becomes:

$$T = \max \left\{ \left[ \frac{75}{(\sigma \times 10^3)} \right]^{\frac{1}{3}} , \left[ \frac{781.25}{(E \times 10^6)} \right]^{\frac{1}{4}} \right\} \quad (7)$$

The two materials considered have the following properties:

<table>
<thead>
<tr>
<th></th>
<th>$\sigma$ (MPa)</th>
<th>$E$ (GPa)</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$C_r$ ($/kg$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon Steel</td>
<td>345</td>
<td>207</td>
<td>7095</td>
<td>1.00</td>
</tr>
<tr>
<td>Aluminum (2024)</td>
<td>455</td>
<td>72.4</td>
<td>2774</td>
<td>3.50</td>
</tr>
</tbody>
</table>

The values for $T$ would be:

$$T_{\text{carbon steel}} = \max [0.0601, \ 0.0441] = 0.0601\text{m} = 6.01\text{ cm}$$

$$T_{\text{aluminum}} = \max [0.0548, \ 0.0573] = 0.0573\text{m} = 5.73\text{ cm}$$

Note that for the carbon steel, the binding constraint is the strength or load and for the alloyed aluminum, the binding constraint is the stiffness or deflection. It often takes considerable time to develop problems with the appropriate inputs of load, length and deflection to assure that the binding constraints are different for different materials. The cost for the carbon steel and aluminum structures would be:

$$C_{\text{carbon steel}} = 1.00\$/kg \times 7095\text{ kg/m}^3 \times (0.0601\text{ m})^2 \times (0.25\text{ m}) = $ 6.42$$

$$C_{\text{aluminum}} = 3.50\$/kg \times 2774\text{ kg/m}^3 \times (0.0573\text{ m})^2 \times (0.25\text{ m}) = $ 7.97$$

If one wants to focus upon weight, the values would be:

$$W_{\text{carbon steel}} = 7095\text{ kg/m}^3 \times (0.0601\text{ m})^2 \times (0.25\text{ m}) = 6.41\text{ kg}$$

$$W_{\text{aluminum}} = 2774\text{ kg/m}^3 \times (0.0573\text{ m})^2 \times (0.25\text{ m}) = 2.28\text{ kg}$$

The weight of the aluminum structure is considerably less than that of the carbon steel, but the higher cost makes it less attractive on an economic basis. If there was a 0.50 $/kg penalty
cost for every kg over 2.28 kg, the cost of the carbon steel would increase by \((6.41 - 2.28) \times 0.5 = 2.07\) which would make carbon steel (\$ 8.49 with penalty) more expensive than the aluminum alloy (\$ 7.97). In many instances, such as in spacecraft, airplanes, and cars there are weight penalty costs. If a higher strength carbon steel was used, its dimensions and weight could be reduced. However, for the aluminum alloy, stiffness was the binding constraint and the modulus of elasticity is relatively constant for materials and changes little with alloying or heat treatment.

**The Computer Model**

The modules of the program and the relationships between the modules are shown in Figure 1. The program was written in Microsoft Excel and the macros are coded in VBA (Visual Basic for Application). A copy of the program was e-mailed to all the students and they could either modify the program for their additional shapes or use any other program to solve the shape(s) which were not initially included in the program. The load and support modules determine the force and moment relationships and the shape module determines the centroid and moment of inertia expressions. Expressions for moment and deflection (Appendix 1) for different shapes were built into the computer model. The parameter selection module requires that certain parameters be fixed so that only one variable is determined. The parameters may be specified as a specific value or as a ratio. For example in the evaluation of a rectangular shape the width or height can be specified and the other value would be determined. A ratio of height-to-width could be specified and then one of the values would be solved for and the other determined by the ratio.

The total cost is the sum of the material cost, processing cost, and penalty cost. The material cost is determined by the weight of the material and the cost per unit weight. The penalty cost is based upon the amount of weight above the desired weight level. The desired level and the penalty per unit cost must be specified. The processing cost expression is a theoretical expression based upon the material properties and the shape of the material being processed (Appendix 2). The processing costs can be approximated by the expression:

\[
C_p = \left(\frac{P}{A_c}\right) \times \left(\frac{T_{mp}}{1800}\right) \times CW_f \times HT_f \times R_{pc} \times L \times A_c \times N_I
\]  

(8)

Where:

- \(P\) = total perimeter of cross-section (in, m)
- \(A_c\) = cross-sectional area (in², m²)
- \(T_{mp}\) = melting point of alloy (°K)
- \(CW_f\) = cold work factor = 1.25 if material is cold worked, otherwise use 1.0
- \(HT_f\) = heat treating factor = 1.35 if material is heat treated, otherwise use 1.0
- \(N_I\) = hollow internal section factor = 0.70 if material has a hollow internal section, otherwise use 1.0
- \(L\) = part length (in, m)
- \(R_{pc}\) = relative processing cost = 0.03 $/in², 46.5 $/m²
Figure 1. Modules Used For Computer Model for Economic Design Evaluation

**User Interface**

**Input Module**
- Input the values for required deflection, beam length, penalty cost etc.

**Load Selection Module**
- Select the type of loading (and input the value) from the following list:
  A. Single point load
    - Center load
    - End load
  B. Uniform load
  C. Input the value of design load.

**Support Selection Module**
- Select the type of support(s) on the ends from the following:
  A. Simply supported ends
  B. Fixed ends
  C. Cantilever support

**Shape Selection Module**
- Select the cross-sectional shape from the following:
  A. Square
  B. Rectangular
  C. Circular
  D. I-beam
  E. Hollow-box beam
  F. Equilateral triangle

**Parameter Selection Module**
- Specify the following shape parameters (if necessary):
  A. Height
  B. Width
  C. Thickness(es)

**Materials Database Module**
- This module has the values for the following parameters:
  A. Young's modulus
  B. Yield strength
  C. Density
  D. Melting point
  E. Cost per unit weight

**Load Conditions and Shape Parameters**

**Material Properties**

**Mechanical, Material, Cost**

**The Model**

**Analysis Module**
- Computer model analysis

**Results Module**
- The results module includes the following information:
  A. Design parameter value (strength constraint)
  B. Design parameter value (stiffness constraint)
  C. Selected design parameter
  D. Weight of the chosen material
  E. Materials cost, Penalty cost, Processing cost, Total cost
  F. Graphical display of results
The input and results for the example problem are presented in Table 1 and Figure 2. The different cost components are illustrated in Figure 2 to indicate the effect that the penalty and processing costs have on the total costs. The user can eliminate the penalty costs or processing costs by inserting values of zero for the penalty cost and the relative processing cost. The project assigned to the students is in Appendix 3 and the results are illustrated in Table 1 and Figure 2 for the two materials considered in testing the program.

Table 1. Input and Results for Illustrative Example Problem

Module Inputs

<table>
<thead>
<tr>
<th>Loading &amp; Shape Parameters</th>
<th>Design Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of Loading</td>
<td>Deflection Limit</td>
</tr>
<tr>
<td>Single Point Load</td>
<td>0.001 m</td>
</tr>
<tr>
<td>Center Load</td>
<td>Beam Length</td>
</tr>
<tr>
<td>Design Load</td>
<td>0.250 m</td>
</tr>
<tr>
<td>200 kN</td>
<td>Penalty Cost</td>
</tr>
<tr>
<td>Type of Supports</td>
<td>0.50 $/kg</td>
</tr>
<tr>
<td>Simply Supported Ends</td>
<td>Penalty Limit</td>
</tr>
<tr>
<td>Shape Selection</td>
<td>2.28 kg</td>
</tr>
<tr>
<td>Square</td>
<td></td>
</tr>
<tr>
<td>Parameter Selection</td>
<td></td>
</tr>
<tr>
<td>None Required</td>
<td></td>
</tr>
</tbody>
</table>

Process Parameters

| Cold Work Factor | 1.0 |
| Heat Treating Factor | 1.0 |
| Hollow Internal Section Factor | 1.0 |
| Relative Processing Cost | 46.5 $/m² |

<table>
<thead>
<tr>
<th>Materials Considered</th>
<th>Material Properties</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Young’s Modulus GPa</td>
<td>207</td>
<td>72.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Yield Strength MPa</td>
<td>345</td>
<td>455</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Density kg/m³</td>
<td>7095</td>
<td>2774</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Melting Point °K</td>
<td>1800</td>
<td>900</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Relative Cost $/kg</td>
<td>1.00</td>
<td>3.50</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Carbon Steel         | 0.0441| 0.0601| 0.0601| 0.00362| 6.42| 6.42| 2.07| 2.80| 11.28 |
| Aluminum (2024)      | 0.0573| 0.0548| 0.0573| 0.00328| 7.97| 2.28| 0.00| 1.33| 9.30  |

Module Results
Alternative Approach

The computer program, Cambridge Engineering Selector 4(CES4), which is marketed by Granta Design Limited was considered for use but was not selected for several reasons. The students have had no previous exposure to the CES4 program in their materials and mechanics of materials courses. Serious study and time are required to utilize the program and insufficient time was available in the 2 credit course which primarily focuses on manufacturing processes. The second issue is that the course has a large number of students in the spring semester, ranging 60 to 100 students, and it is rather difficult to instruct such a large number of students in a short time. The approach used, where the students calculate the final dimensions of the structure in the program, is preferred over the CES4 selection approach based upon material property ratios. If a follow-up elective course is offered, the CES4 package would probably be used in conjunction with the program developed as the CES4 has many additional design considerations.

Cost Analysis of Materials

![Graphical Display of Cost Analysis of Materials from Results Module](image)

**Figure 2. Graphical Display of Cost Analysis of Materials from Results Module**

Program Application

The program was used during the Spring Semester of 2005 in the basic manufacturing processes course, which initially had 100 students enrolled. The students will use the program and then will be requested to develop a solution for a shape which is not in the program to illustrate they understand the process. The program is limited to relatively simple loading conditions and shapes, but it does indicate the importance of the integration of materials, shape, loading conditions and material costs upon design. The results regarding the relative success (or
lack thereof) of the program will be presented at the ASEE Annual Meeting in 2005. The evaluation questionnaire prepared for assessment of the program is in Appendix 4.

**Bibliographic Information**


**Biographical Information**

ROBERT C CREESE is a professor in the Industrial and Management Systems Engineering Department in the College of Engineering and Mineral Resources at West Virginia University in Morgantown, West Virginia. He obtained his BS, MS, and Ph.D. degrees from The Pennsylvania State University, The University of California-Berkeley, and The Pennsylvania State University. He is a member of ASEE and also a member of AACE International, ASM, AWS, AIME, ISPA, SCEA, AFS, and SME.

DEEPAK GUPTA is a graduate student in the Industrial and Management Systems Engineering Department in the College of Engineering and Mineral Resources at West Virginia University in Morgantown, West Virginia. He obtained his BS degree from the University of Roorkee, now IIT – Roorkee, and is working on his MS and Ph.D. degrees at WVU. He is a member of AACE International, IIE, INFORMS.
Appendix 1: Moment and deflection equation for different cases

<table>
<thead>
<tr>
<th>Load Profile</th>
<th>Center Load</th>
<th>End Load</th>
<th>Uniform Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simply supported</td>
<td>$M = \frac{w*l}{4}$</td>
<td>$\delta = \frac{w<em>l^3}{48</em>E*I}$</td>
<td>$M = \frac{w*l^2}{8}$</td>
</tr>
<tr>
<td>Fixed ends</td>
<td>$M = \frac{w*l}{8}$</td>
<td>$\delta = \frac{w<em>l^3}{192</em>E*I}$</td>
<td>$M = \frac{w*l^2}{24}$</td>
</tr>
<tr>
<td>Cantilever</td>
<td>$M = \frac{w*l}{2}$</td>
<td>$\delta = \frac{5<em>w</em>l^3}{48<em>E</em>I}$</td>
<td>$M = \frac{w*l^2}{2}$</td>
</tr>
</tbody>
</table>

Appendix 2: Processing Cost Equation

Equation 8 was developed based on the consideration that the shapes would be formed by rolling, extrusion, or similar processing. No suitable processing cost equation was found in the literature. The major factors considered in the equation development were the material properties of the material being processed and the tooling costs for the shape being processed. The factors considered were:

1) Perimeter – The larger the perimeter, the more machining of the tooling and the costs were assumed to increase proportionally.
2) Shape factor = perimeter/cross-sectional area. As the cross-sectional area increased, the shape factor would reduce, but larger cross-sectional areas would increase the size of the tooling and the net effect of area was assumed to be negligible.
3) Temperature – Higher melting point temperatures would result in higher processing temperatures and more expensive tooling.
4) If processing was to include heat treatment or cold work, the processing costs would increase.
5) Internal sections increase the processing costs, but since the perimeter increases with the use of an internal section, the adjustment factor selected was 0.7 after some test runs. Since the hollow sections tended to result in thin-walled structures, the perimeter would nearly double and thus the net factor for a hollow section was estimated to be in the range of 1.2 to 1.4.
6) The increased part volume, represented by the product of cross-sectional area and length would increase the processing costs. The increased length would increase the processing time.
7) Equation 8 will be modified when a better expression is obtained.

Equation 8

\[ C_p = \left( \frac{P}{A_c} \right) \times \left( \frac{T_{mp}}{1800} \right) \times CW_f \times HT_f \times R_{pc} \times L \times A_c \times N_l \]

Where:
- \( P \) = total perimeter of cross-section (in, m)
- \( A_c \) = cross-sectional area (in\(^2\), \( m^2 \))
- \( T_{mp} \) = melting point of alloy (degrees K)
- \( CW_f \) = cold work factor = 1.25 if material is cold worked, otherwise use 1.0
- \( HT_f \) = heat treating factor = 1.35 if material is heat treated, otherwise use 1.0
- \( N_l \) = hollow internal section factor = 0.70 if material has a hollow internal section, otherwise use 1.0
- \( L \) = part length (in, m)
- \( R_{pc} \) = relative processing cost = 0.03 $/in\(^2\), 46.5 $/m\(^2\)

Notes for Equation 8
1) \( P/A_c \) = shape factor
2) \( T_{mp}/1800 \) = temperature processing factor (materials with higher melting points are more difficult to process as the hot working temperatures are higher)
3) \( L \times A_c \) = volume factor
4) Some material costs already include the processing costs such as the factors for cold working or heat treatment. If these processing costs are in the material costs, then they do not need to be included in the processing costs.
Appendix 3: Problem for Spring Semester 2005

Design Problem with Alternative Materials and Shape Variations
USA Version (metric version)

The CEO of RCC, LTD is entering the ladder business and wants the design team to design rungs for the ladder. The design engineer has estimated that the rungs will need to support a dead load of 400 lbs. (182 kg) with a maximum deflection of 0.100 inches (2.54 mm). The rungs have a length of 16 inches (40 cm) and the height between rungs is 12 1/8 inches (30.3 cm). The rungs have a cost penalty of $0.50 per pound for every pound over 0.40 pounds as light weight is greatly desired for extension ladders. Assume the extension ladder will have 12 rungs and the sides will weigh 4 times the total weight of the rungs.

The initial materials and the material properties being considered are:

<table>
<thead>
<tr>
<th>Young’s Modulus (E) (psi x 10^6)</th>
<th>Yield Strength (σ) (psi x 10^3)</th>
<th>Density (ρ) (lb/in^3)</th>
<th>Cost ($/lb)</th>
<th>Melting Point (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon Steel</td>
<td>30</td>
<td>0.28</td>
<td>0.80</td>
<td>1800</td>
</tr>
<tr>
<td>Aluminum</td>
<td>10</td>
<td>0.10</td>
<td>2.20</td>
<td>900</td>
</tr>
</tbody>
</table>

The design considerations for strength and stiffness for center single point loading with fixed ends are:

For Strength: \[ \sigma = \frac{PLc}{8I} \]
For Stiffness: \[ \delta = \frac{PL^2}{192EI} \]
For Square Cross-Section: \[ b = h \]

Where:
\[ c = \frac{h}{2} \quad \text{and} \quad I = \frac{1}{12} bh^3 = \frac{1}{12}h^4 \]
\[ \delta = \text{maximum deflection limit (0.10 inches, 2.54 mm)} \]
\[ L = \text{rung length (16 inches, 40 cm)} \]
\[ P = \text{design load (400 lb., 182 kg)} \]
\[ E = \text{Young’s Modulus} \]
\[ \sigma = \text{Yield Strength} \]
\[ b = \text{rung width} \]
\[ h = \text{rung height thickness} \]

A sketch of the ladder loading is:

![Sketch of ladder loading](image-url)
The total cost is the sum of the material cost, processing cost, and the cost penalty. The processing can be approximated by the expression:

\[ C_p = (P/A_c) \times \left( \frac{T_{mp}}{1800} \right) \times CW_f \times HT_f \times R_{pc} \times L \times A_c \times N_l \]

Where:
- \( P \) = total Perimeter of Cross-section (in)
- \( A_c \) = cross-sectional area (in\(^2\))
- \( T_{mp} \) = melting point of alloy (degrees K)
- \( CW_f \) = cold work factor = 1.25 if material is cold worked, otherwise use 1.0
- \( HT_f \) = heat treating factor = 1.35 if material is heat treated, otherwise use 1.0
- \( N_l \) = hollow internal section factor = 0.70 if material has a hollow internal section, otherwise use 1.0
- \( L \) = part length (in)
- \( R_{pc} \) = relative processing cost = 0.03 $/in\(^2\)

Note:
1) \( PC/A_c \) = shape factor
2) \( T_{mp}/1800 \) = temperature processing factor (materials with higher melting points are more difficult to process as the hot working temperatures are higher)
3) \( L \times A_c \) = volume factor
4) Some material costs already include the processing costs and the factors for cold working or heat treatment. If these are in the costs, then they do not need to be included again.

Base Solution – Shape is square cross-section; therefore \( I = (1/12) \times b h^3 = (1/12) \times h^4 \) as \( b = h \).

Deflection Constraint
\[ \delta = \frac{PL^3}{192EI} = \delta = \frac{PL^3}{192E(h^4/12)} = \frac{PL^3}{16Eh^4} \]

Thus,
\[ h = \left( \frac{PL^3}{16E\delta} \right)^{0.25} = \left( \frac{400 \times 16^3}{16 \times E \times 0.10} \right)^{0.25} = (1.024 \times 10^6/E)^{1/4} \]

Strength Constraint,
\[ \sigma = \frac{Mc}{I} = \frac{(PLc/8l)}{((PL \times h/2)/(8 \times h^4/12))} = ((3PL)/(4h^3)) \]

Thus,
\[ h = \left( \frac{3PL}{4\sigma} \right)^{0.33} = \left( \frac{3 \times 400 \times 16}{4 \times \sigma} \right)^{0.33} = (4.8 \times 10^3 /\sigma)^{1/3} \]

Thus to satisfy both constraints,
\[ h = \max \left[ (1.024 \times 10^6/E)^{1/4}, (4.8 \times 10^3 /\sigma)^{1/3} \right] \]
for carbon steel $E = 30 \times 10^6$ psi and $\sigma = 50 \times 10^3$ psi and thus

$$h_{\text{steel}} = \max[0.430 \text{ in}, 0.458 \text{ in}] = 0.458 \text{ in} \quad \text{(strength(load) is the binding constraint)}$$

for aluminum, $E = 10 \times 10^6$ psi and $\sigma = 35 \times 10^3$ psi and thus

$$h_{\text{Al}} = \max[0.566 \text{ in}, 0.516 \text{ in}] = 0.566 \text{ in} \quad \text{(stiffness(deflection) is the binding constraint)}$$

**Cost Totals**

**Material Costs**

$$C = C_R x \rho x A x L$$

$$C_{\text{steel}} = 0.8 \$/lb x 0.28 \text{ lb/in}^3 x (0.458 \text{ in})^2 x 16 \text{ in} = 0.752$$

$$W_{\text{steel}} = 0.28 \text{ lb/in}^3 x (0.458 \text{ in})^2 x 16 \text{ in} = 0.940 \text{ lb}$$

$$C_{\text{Al}} = 2.2 \$/lb x 0.10 \text{ lb/in}^3 x (0.566 \text{ in})^2 x 16 \text{ in} = 1.128$$

$$W_{\text{Al}} = 0.10 \text{ lb/in}^3 x (0.566 \text{ in})^2 x 16 \text{ in} = 0.514 \text{ lb}$$

**Penalty Cost (for weight above 0.40 lbs @ 0.50/lb)**

$$\text{Steel} = (0.94 - 0.40) x 0.50 = 0.270$$

$$\text{Al} = (0.514 - 0.40) x 0.50 = 0.057$$

**Processing Costs**

$$C_p = (P/A_c) x (T_{mp} / 1800) x CW_f x HT_f x R_{pc} x L x A_c x N_l$$

$$C_{p-\text{steel}} = (4 x 0.458/0.458^2) x (1800 / 1800) x 1.0 x 1.0 x 0.03 x 16 x 0.458^2 x 1.0 = 0.879$$

$$C_{p-\text{Al}} = (4 x 0.566/0.566^2) x (900 / 1800) x 1.0 x 1.0 x 0.03 x 16 x 0.566^2 x 1.0 = 0.543$$

**Total Cost**

$$\text{Steel} = 0.752 + 0.270 + 0.879 = 1.901$$

$$\text{Al} = 1.128 + 0.057 + 0.543 = 1.728$$

In this case, aluminum has a total cost advantage (about $0.17). However, carbon steel has a material cost advantage (about $0.38). If the penalty costs were not included, carbon steel would have a slight cost advantage (about $0.04). Thus, one can understand the problems that occur in selecting materials, and that total cost is desired. The values for penalty costs and processing costs must be accurate to have accurate total costs.
Estimated total weight of the ladder:

- **Steel**: \(12 \text{ rungs} \times 0.94 \text{ lb/rung} \times 4 \text{ factor} = 45.1 \text{ lbs}\)
- **Aluminum**: \(12 \text{ rungs} \times 0.514 \text{ lb/rung} \times 4 \text{ factor} = 24.7 \text{ lbs}\)

Aluminum has a definite weight advantage.

The Project Assignment

These costs are unacceptable to the CEO and he wants to see how much you can lower the costs without decreasing the load capacity or increasing the deflection limit, as more movement would be noticeable.

The CEO has been advised by his lawyer to increase the safety factor and increase the load to 800 pounds (364 kg) and his ergonomics specialist has advised him to make the width of the rung at least 1.5 inches (3.81 cm).

Your team is to consider other materials [those in Table 6.2(increase the costs by 50%) and select at least one additional material] You are also to consider the following shapes:

1) Rectangular – \(\text{width} = 2 \times \text{thickness}\)
   \(\text{width} = 3 \times \text{thickness}\)
2) Equilateral triangle (point down)
3) I-beam with flange thickness 1/8 inch (3.18 mm), I thickness = \(\frac{1}{4}\) inch (6.35 mm)
4) Hollow box beam (minimum thickness 0.080 inches or 2.03 mm) (determine height as width is specified)
5) Hollow Circle (diameter = 1.5 inches, minimum thickness is 0.080 inches (2.03 mm))
6) A flange shape (minimum thickness is 0.080 inches, height is variable)
7) Optional – one additional shape of your selection

Minimum additional design changes to be considered:
1) Making a wide-body ladder, that is increasing the rung length from 16 inches to 20 inches to meet the larger body and keeping the same constraints.
2) Increase the design load to 1,600-lb (727.3-kg)
3) Increase the maximum deflection to 0.200 inches (5.08 mm)
4) Decrease the maximum deflection to 0.050 inches (1.27 mm)

You may use the computer program developed by Mr. Gupta for many of the cases, but you may need to develop a program for the flange shape and hollow circular shape.
The output should include the dimensions of the rung and the parameter, such as thickness, for both constraints and the value selected. Put the results into tables in the report. In addition, any derivations for the formulas for the additional shapes should be included in the report.

The selection should consider the best material and the three next best alternatives. This should be done for total cost, material only cost, weight, and volume. If you use a combined weighing index, explain the index and why you used it.

Super students only – what would be the difference if the loading was considered as uniform loading instead of single point loading? Consider this for only your best scenario.

Prepare a report recommending your selection of which material and shape would be used. Include the cost savings of your design versus the original square shape.
Appendix 4: Assessment Questionnaire for Computer Program and Project.

To assist in understanding of the interaction of the material properties, mechanical properties, and the product shape upon the total material cost, the computer program was developed to reduce the calculation discrepancy in evaluating numerous shapes. You were requested to evaluate one or two shapes not in the compute program to illustrate that you understand the calculation process. The following questions are asked to assess the value of the software. Please use a scale of “1” to “5”, “1” being “Completely Disagree” and “5” being “Completely Agree”.

A. The program is better than working out the details on your own.
   1  2  3  4  5

B. The program was easy to understand and work with compared to other software program you have used.
   1  2  3  4  5

C. The program lets the student apply their skills and improve it further.
   1  2  3  4  5

D. Format in which the results are displayed (tabular and graphical) is good.
   1  2  3  4  5

E. The program should be given to all future IENG 302 students since it saves time and effort in solving the project problems.
   1  2  3  4  5

F. Sufficient technical support was available to all the students whenever they needed it.
   1  2  3  4  5

G. The current team size of 2-3 students is optimal for the project.
   1  2  3  4  5

H. What would be the most important improvement you would make to the program?

I. What other suggestions for improvement would you recommend for the computer software or project assignment?

Other Comments/Recommendations: