AC 2012-3138: WHEN TO START COLLECTING SOCIAL SECURITY: DESIGNING A CASE STUDY

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Abstract

Engineering economy courses often include case studies as a bridge between the simplifications of end-of-chapter problems and real world engineering economic analysis. Unfortunately most of those case studies must translate the task of searching out the data into one of simply reading the case description. While important and irrelevant data can be included, the basic steps in analyzing the problem are often obvious.

In contrast, it is possible to use the question of when to start collecting social security as a case where students must search out the data and decide how to structure the analysis. Moreover, because of the myriad details and situations, instructors can match the complexity of the assignment to their students’ maturity and their course’s time constraints. Instructors can also assign variations to ensure each assignment has appropriate difficulty, but that results can be generalized over the class to illustrate the range of possibilities.

Many engineers, faculty, and the parents and grandparents of our students must answer the question of whether to start collecting reduced Social Security payments at age 62, wait until age 66 for regular payments, or to delay until later for enhanced payments. Thus, the case can be made relevant for most students by framing this as: “Your grandparents or parents need advice.” While the U.S. Social Security system is used here to illustrate possible case studies and results, the approach is applicable for similar problems with other defined benefit retirement systems.

There are sources that do properly include the time value of money, but the publications of the Social Security Administration and many others focus on the dollar difference in monthly payments and do not even touch on the time value of money. Thus, all case possibilities include how to properly include the time value of money. Then the instructor can choose how to limit the boundaries of student work.

This paper discusses possible choices for case studies and example analyses that might be done for those choices. For example, “Obviously the interest rate and the predicted age at death are both uncertain. Choices need to be analyzed for a variety of interest rates and expected lives. Present the results in a format that makes sense to someone who understands the meaning of present worth and rate of return, but who may not be able to calculate them.”

This paper also includes example analyses of how the attractiveness of the early collect option may be eliminated by rules on income from continuing employment and on benefits from a second pension. However, the focus is pedagogical. What data is needed, where can it be found, what economic measures are valid and easiest to understand, how can different choices be compared, and how should the results be communicated? The paper closes with an overview of deeper levels of analysis that may be suitable for more advanced courses.
Introduction

The U.S. Social Security program is one of the largest government programs in the world, ranking as the third largest expenditure of the federal budget in Fiscal 2010. Social Security accounted for 20.3% of the budget, compared to 23.6% for discretionary defense and 20.8% for Medicare/Medicaid. Thus, analysis of the program is a full-time job for many, as well as being the subject of ongoing political discourse.

Nevertheless, at the level of personal decision-making much of the available information does not seem to properly consider the time value of money. Since how to properly consider that time value is the subject of engineering economy courses, the topic is a suitable one for a case study. More importantly, like a real world problem, analyzing this case study can require students to search out the needed information from many possible sources, read and understand detailed rules, make reasoned judgments about which factors are critical and about which assumptions should be made, and select effective presentation approaches.

So instructors need only construct an appropriate motivating description that is accompanied by a description of what they expect to see in a solution. To match their pedagogical goals, instructors can simplify the problem to one gender or to the evaluation of one decision choice, or instructors can add the consideration of expected mortality to force development of a recommended decision. Instructors can even have students analyze this in successive rounds of increasing detail and complexity.

While there are many different possible levels of analysis, the problem does permit construction of a right answer at each level—which is very reassuring to students who are not used to case analysis. It can also be very helpful in grading the work of many student groups.

After summarizing some of the literature threads, this presentation is organized to support use by other faculty in the classroom. It starts at the most basic level that is better described as a mini-case, continues through a case that might be assigned for the introductory undergraduate course, continues through a more advanced consideration of the uncertainty of mortality, and closes with descriptions of further work that might be included by teams in a graduate course.

Literature Review

There is a substantial literature analyzing Social Security as a system, and some of it directly addresses the timing of initial payments to recipients. There are many sources of information on the details of regulations and calculations, but the most authoritative and accessible source is clearly the website of the Social Security Administration (SSA). Not as authoritative, but probably as accurate and certainly more readable are the many Wikipedia entries. There are also many refereed publications and non-refereed sources.

Of primary interest here is the literature concerning the individual decision, which has a long history and continuing interest. Non-refereed publications in particular can provide models to students of which assumptions may make the most sense and of how to present the results to those facing their own decision. However, most of these focus on the monthly payment...
level or the total of the monthly payments to be received not on an economically valid measure of the total expected income stream—exceptions, of course, exist.\textsuperscript{14,15}

The future age at death for a recipient is the key unknown in the decision of when to start collecting. The estimated benefit calculator on the social security website\textsuperscript{16} provides added life expectancies by gender and birth date. There are also refereed publications on the problem of estimating remaining life.\textsuperscript{17,18}

**Retirement versus Beginning to Collect Social Security**

When framing the case, we suggest that it is important to emphasize that retirement is neither necessary nor sufficient to answer the question of when to begin collecting social security. Economic analysis considering the time value of money is best focused on a more limited question about when to being collecting benefits.

The lifestyle choices that are the focus of retirement may or may not be heavily constrained by economics. However, the lifestyle choices driven by individual values and situations seem to be well beyond the ability of anyone to model. What are the trade-offs between work that may be fulfilling or exhausting and boring, time to travel the world or to live near or visit grandchildren, time to focus on a favorite or new hobby, time to make a difference by volunteering, care-taking requirements for an ailing or aging spouse or parent, etc.? All of these are difficult to value for an individual and impossible to generalize for a population. In addition, some people choose to continue to work past the normal Social Security retirement age, and draw benefits while still working full time. Thus, a case study that addresses the “retirement” decision must be for an individual or family, and its non-economic features will dominate the economic analysis of the time value of money which will often be small or non-existent.

While the word *retirement* can be avoided in a problem statement, its pervasive use in all of the literature (especially within SSA information) means that it will almost certainly be part of the discussion of the case.

A more realistic goal for a case study in engineering economy is to model the economic choice for individuals where the *retirement* decision is separable from the “time to collect” decision. So, for a rigorous formulation the options of when to begin collecting benefits only apply to individuals with sufficient savings, investments, or other sources of income to support themselves if *now* is not the time to collect. If the decision to retire requires that social security be started immediately in order to pay for food, shelter, etc. then non-economic retirement trade-offs should probably dominate decision-making.

The ability to separate the retirement and time to collect decisions does not imply that the decisions are independent. For example, as will be analyzed later, continuing in a well-paid job implies that the decision to wait until the normal retirement age of 66 is typically a no-brainer. Similarly, if an individual has been diagnosed with a terminal illness then typically the decision to begin collecting Social Security benefits as soon as possible is clearly dominant, and the decision of when to stop work may depend on the incremental economic value of that additional work.
This can be the first evaluation point for student work. Did they identify and describe when their results can and cannot be used? Have students stated their assumptions about income levels, other pensions, taxes, and covered or uncovered spouses?

**Defining the Case**

The assumptions discussed above are closely linked to how the instructor chooses to define the case. An individual “base” case can be clearly specified and tightly limited, or the case description may require consideration of a broad range of possibilities. Fundamentally, the selection of reasonable student values is guided by the instructor’s choices of how simple or comprehensive to make the case. The range of complexity is suggested by these possibilities.

- Use PW to help a single, 62 year old, male with an AIME of $3342 determine the value of starting to collect benefits now. His interest rate for the time value of money is 3%.
  - Requires students to research the data (including finding out what an AIME is) and to structure the problem. This is a simple PW calculation with different results for a range of ages at death.
  - If broadened to include whether he should start benefits now, then students might compare starting at 62 with age 63 or with age 66 or with a range of ages.
- When should a 60 year old female who qualifies for social security begin collecting benefits?
  - This case requires students to consider whether the value of the AIME matters, to consider a range of interest rates, to consider ages 62 to 70 as possible starting ages for collecting benefits, to choose a measure of economic value, and to decide how to present the results.
  - Specifying an interest rate, a minimum or maximum age at death, possible retirement ages to consider, or the economic measure to use would each simplify this general case. For example, specifying PW or IRR precludes the student from using the very commonly used total of benefits received.
  - This could present results for a range of ages at death or it could use mortality data to make recommendations.

The following presentation generally uses the language and acronyms of the official Social Security website, and it relies on data from that website. The choice of a base case situation may or may not be generally applicable, and it can simplify or complicate the analysis. Thus, a second evaluation point for student work is the credibility of their data sources and the quality of their base case assumptions. Some of these assumptions will come directly from the instructor’s case statement, other assumptions will be guided by that statement, and yet other assumptions may be addressed or omitted simply due to the student’s diligence or ignorance.

For clarity let us start with the three major components, and the official SSA terminology, used in calculating the monthly retirement benefit:

- Average indexed monthly earnings (AIME); the average monthly income subject to Social Security taxes during the previous 35 years having the highest income
- Primary insurance amount (PIA); the monthly Social Security payment, or benefit at the normal retirement age
Retirement age or time to begin collecting benefits

Students may define their base case by asking a parent or grandparent to use the SSA’s benefits estimator\(^\text{16}\). This will automatically produce estimates of the PIA for a normal retirement age and estimates for early retirement at 62 and delayed retirement at 70.

We suggest that computation of the AIME for an individual should not be part of a student case study. This requires a 35 year earnings history and research into the detailed values for properly indexing each year. Note that the indexing adjusts for inflation, but not for a salary trajectory from entry-level to higher-paying positions. Since this value essentially becomes a constant that scales all results, highly accurate estimates are not needed. Our analysis starts with the 2010 national average wage index of $41,673.83.\(^\text{18}\) The average for men ($46,800 in 2009) is higher and the average for women ($35,000 in 2009) is lower.\(^\text{19}\) This is because women tend to have more breaks in employment and have lower average earnings than men.\(^\text{20}\) Our value for AIME divides the average annual wage index by 12 to compute a monthly $3473. For this example, we will reduce the AIME to $3000 to adjust for the possibility of a few missing years and a salary trajectory. The rounded value also emphasizes the approximations involved.

As noted by a reviewer, the selection of an AIME is a good opportunity to evaluate the success of a student team in finding good data and making reasonable assumptions.

If the SSA’s benefit estimator is used, the 4 year difference between early and normal retirement may significantly change the AIME computation. If a person has less than 35 years with significant income subject to Federal Insurance Contributions Act (FICA) tax, then delaying retirement may raise their AIME significantly. For example, a state university professor may have many years covered by a state pension plan rather than social security and an individual with significant child-care gaps in their employment record may raise their AIMEs with FICA taxed income prior to retirement. To avoid this complexity, our computations assume that added years of employment do not change the AIME.

The computation of the normal monthly retirement benefit, the PIA, relies on a formula which contains “bend points” that increase with inflation. For 2012, the PIA equals 90% of the AIME up to 1\(^{\text{st}}\) bend point of $767, 32% of the incremental AIME up to the 2\(^{\text{nd}}\) bend point of $4624, and 15% of the incremental AIME above that point.

\[
\text{If AIME} < 767, \quad \text{PIA} = (0.90)(\text{AIME}) \\
\text{If} 767 < \text{AIME} < 4624, \quad \text{PIA} = (0.90)(767) + (0.32)(\text{AIME} - 767) \quad (1) \\
\text{If AIME} > 4624, \quad \text{PIA} = (0.90)(767) + (0.32)(4624 - 767) + (0.15)(\text{AIME} - 4624)
\]

Note: the schedule is progressive and this is why the social security benefit may nearly replace pre-retirement income for low AIMEs, but not for higher-level AIMEs. For our assumed $3000 value for AIME, the PIA is $1405 \(= .90 \times 767 + .32 \times (3000 - 767)\).

To at least partially account for increasing expected life spans and for questions about solvency, there have been and there are future scheduled increases in the “normal” retirement age. However, since those whose birth years are from 1943 – 1954 have a normal retirement at 66
between 2009 and 2020, the most appropriate base case is simply 66 years. With this normal retirement age, early retirement at 62 results in a 25% reduction from the PIA. Delayed retirement at age 70 increases the monthly benefit to 132% of the PIA. Note that these reductions and increases can either be provided to the students, or students can be expected to find the values through a good search. The former is more like a mini-case and the latter is more like the real world.

Values for other retirement ages and other birth years are clearly tabulated in “Effect of Early or Delayed Retirement on Retirement Benefits.” However, we suggest that those minor variations should be ignored. The retirement year bracket of 2009 to 2020 is both current and wide. Three choices (early, normal, and delayed) provide clarity and using 62 and 70 rather than 63, 64, 65, 67, etc. maximizes the differences between the choices.

This is the type of decision that is part of the structure of how to analyze the case. Instructors can direct student choice of ages to be considered or leave it open to student choice. Note that if students consider all age possibilities, then it is more likely that they’ll use an incremental or marginal analysis approach—which could be very different than what is presented here.

Thus, the assumed monthly benefit at age 62 is $1053.75 (= .75×1405), at 66 it is $1405, and at 70 it is $1854.60 (= 1.32×1405).

**Which Measure of Economic Value?**

As noted earlier, there are at least three measures of economic value that are used in evaluating when to start collecting benefits. The most common in the popular literature is the total amount of benefits received. We hope that most students in an engineering economy course would choose present worth or IRR (the latter is likely to be the incremental IRR when comparing choices.

Each of these measures depends on the assumed or calculated level of monthly benefits. Estimates of future benefits for individuals were historically mailed by the SSA, and they are now available on the agency’s website. These list monthly benefits with retirement ages of 62, 66, and 70. This presentation and many popular press stories focus on the monthly values ($1054, $1405, and $1855 above). The difference in start dates can be part of analyzing the breakeven points in comparing the total dollars to be received under the 3 choices. (The results of using total dollars received can be duplicated by simply assuming that the interest rate is 0% when properly considering the time value of money.)

One potential error that can be made in selecting the interest rate is to use a *market* interest rate that is not adjusted for inflation. Since the PIA is adjusted for the cost of living, it is better to assume that calculations are being done in constant value dollars. If constant value dollars are used, then a *real* interest rate (adjusted for inflation) must be used. In our calculations the assumed real interest rates are treated as nominal annual rates and converted to monthly interest rates for the actual computations.
One easy choice is to compare the present worth of each choice at age 62. One of the more likely student errors is to compute the present worth of each choice at its starting age, rather than allowing a 4 year or 48 month offset for age 66 and an 8 year or 96 month offset for age 70. Equation 2 places the values in the Excel PV function.

\[ PW = PV\left(\frac{rate}{12}, (Age_{Death} - Age_{Start}) \cdot 12, \text{monthly benefit}\right)/(1 + \frac{rate}{12})^{\{0, 48, \text{or} 96\}} \]  

(2)

Without information on remaining expected life, it is not possible to conclude which choice is best—even if the interest rate is known. Rather either tabulated or graphical values must be used to compare the present worths for different values of age at death. Table 1 has the advantage of providing specific numeric values which make it easier to identify errors or different assumptions in student work. Tables have the disadvantage of providing spurious levels of accuracy since the assumptions are not really precisely known. This disadvantage could be minimized by also highlighting those values that are particularly close to the best choice.

Table 1 illustrates that waiting until normal or delayed retirement age does lead to higher monthly benefits, but how many times that benefit is received is even more important. The calculations at 0% are included for comparison with approaches that ignore the time value of money. At 0% age 78 is the breakeven death age for collecting starting at 62 vs. 66, and age 82.5 is the breakeven death age for collecting starting at 66 vs. 70. These ages at death are relatively young for the average individual who reaches 62, which is why analyses in the popular press often conclude that waiting to collect is the best strategy.

The 3% interest rate is a relatively low value that might be chosen in comparison with relatively secure investments considered over a relatively long-term. The larger the interest rate; the longer a recipient must live in order to justify waiting for larger monthly payments. At 3% the green boxes highlighting the best choice have shifted a bit later. Now the breakeven ages at death are 82.06 years for 62 vs. 66 and 89.93 years for 66 vs. 70.

While not included in Table 1, if the 7% real rate that is recommended in OMB A94 for government agencies is used then starting at age 62 is the best choice for all death ages of 114.1 years or less. Furthermore, both starting at age 62 and age 66 dominate a delayed start at 70.

Figure 1 compares the PW values for early, normal, and delayed retirement at a 3% interest rate. This figure also emphasizes that the differences in the economic value of the choices are near zero for intermediate ages at death, are smaller for older ages at death, and are largest for younger ages at death.

Table 1 and Figure 1 suggest that if the appropriate time value of money is 4% or 5% or higher, that in many circumstances the best economic choice may be starting benefits as early as possible. This contrasts with calculations at 0% where the higher values are not discounted.
In using decision aids such as these an individual brings two important pieces of information that are not available to us. One concerns the individual’s health and potential genetic indicators (How long did their grandparents live?). The second is their subjective evaluation of the risk of collecting smaller amounts or nothing due to an earlier death vs. the risk of collecting less in total and having smaller monthly benefits due to starting benefits earlier.

**Considering Mortality Data**

The analysis so far has not considered gender which is at least statistically part of estimating the expected age at death. On average in the U.S., women live longer than men, which will make starting retirement payments later somewhat more attractive to women.
There are at least three approaches to including mortality data. One approach would focus on the impact of the approximately 2 additional expected years for the lives of females of retirement age. A second approach would be to have data arranged so that the probability of receiving a year’s payments would be matched to the level of those payments. This could even be done with monthly intervals. This approach has two principal advantages:

- It makes the details behind the mortality assumption clear—which makes it easier to compare with individual circumstances.
- It allows the computation of the expected value of the PW or IRR, rather than relying on the approximate calculation of a PW or IRR for an expected life. Since later years are less important than earlier years when discounting good measures of economic value are non-linear for different possible lives.

Because another approach is directly supported by the benefits estimator site of SSA and because of the volume of data required for this approach, this analysis uses a third approach—using the expected additional life expectancy at different retirement ages for different birth years for both men and women. This is a simpler approach for calculating total expected social security benefits, but details of the underlying conditional probabilities are hidden in the “black box” of the online tool. Table 2 lists the added life expectancy values for a man or a woman whose birthday is January 2, 1951 with an early retirement at 62 in 2013 and a normal retirement at 66 in 2017. Note that the gender advantage for women is 2.3 years at age 62 and 1.7 years at age 70.

The values in Table 2 can be used as the expected age at death for calculating the economic worth of starting benefits at an early, normal, or delayed age. For females (males) at interest rates below 4.23% (4.05%), waiting until age 70 is the best choice. For interest rates between 4.23% and 5.17% (4.05% and 4.97%), a normal age of 66 is the best choice of when to collect benefits. For interest rates above 5.17% (4.97%) starting benefits as soon as possible is the best choice.

The difference between starting at age 66 and the other two choices is small, thus Figure 2 omits the curves for an age 66 start. For lower interest rates delaying the start of benefits until age 70 has the highest PW, however at higher interest rates starting as soon as possible at age 62 has the highest PW.

### Table 2. U.S. Life Expectancy for Birthdate of January 2, 1951

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<thead>
<tr>
<th></th>
<th>Male</th>
<th></th>
<th>Female</th>
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<tbody>
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<td>Additional Life Expectancy</td>
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<td>Additional Life Expectancy</td>
<td>Estimated Total Years</td>
</tr>
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<td>70</td>
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Other Choices in Matching Case Complexity to Available Time, Student Ability, and Course Goals

Whether one or many cases are used in a course, every course has a “time budget” for every assignment for both faculty members and students. We believe and suggest that student learning has decreasing returns to scale as the case is made more complex. Thus, a basic assignment might only have students compare starting benefits early vs. at the normal age. This requires the student to properly define the problem, to find good sources for the data, to make assumptions and define a base case, to properly incorporate the time value of money, and to present the results. Other students could do the same, but with the comparison of normal vs. delayed benefits, rather than early vs. normal.

For courses where students have sufficient background in probability, the inclusion of mortality risk and expected values is a natural extension. Again, student skills in economic analysis are unlikely to be significantly increased by analyzing both genders, so this is an easy way to assign students slightly different problems.

For students with more case experience, graduate students, or extra credit there are a number of potential additional points that can be analyzed. Several examples are briefly outlined here. We tend to the belief that these extensions, while practically very important and often crucial to individuals, may not add much incremental value to the student’s experience in economic analysis.

- If a person continues to work while starting benefits at age 62, then $1 in benefits is lost for each $2 in earnings over $14,160. In the year that the recipient reaches age 66, then it is $1 for each $3. This dramatically reduces the attractiveness of starting benefits early for individuals with more than a low income level.

- If a person has a substantial pension, and they do not have 30 years of substantial FICA taxable income, then the Windfall Elimination Provision (WEP) can significantly reduce

![Figure 2. PW vs. Interest Rate for Start Benefits at 62 and 70](image_url)
their PIA. If working from age 62 until starting benefits will reach the 30 year requirement, then continuing to work rather than starting to collect can be quite attractive.

- As discussed earlier, some individuals do not have 35 years of significant indexed FICA earnings. Years of childcare and/or employment by the military or other public pension situations may mean that the 35 years have some $0 or very low value years. For these individuals the 4 years between early and normal or between normal and delayed commencement of benefits may be the opportunity to raise their AIME and thus their PIA through FICA covered employment. A limit for this can be estimated by assuming $0 years are replaced by years at the current level of earning. This increases the value of delaying the start of benefits.
  - This can also be the addressed by student assignments where the situation of a specific individual is being analyzed. In that case specific future earnings can be entered into the SSA benefits estimator.
- Social security benefits were not taxable income until 1984. The portion that is taxed is 50% for joint returns with a combined income (= adjusted gross income + nontaxable interest + ½ of your social security benefits) of $32,000 to $44,000, and 85% for joint returns with a combined income of over $44,000. Notice that tax rates also depend on the level of income. For individuals with taxable pensions, their tax situation may not change significantly with retirement. However, for those with no pension other than social security, matching the start-up of benefits to the cessation of employment may minimize these tax consequences. This seems to be a very difficult area to analyze in generalities.
- Benefits can be claimed by spouses based on the PIA of their spouse (typically at 50% of the level). Since spouses may have very different earning levels and lifetime patterns, this is yet another example of a situation that is more easily analyzed for specific individuals than in generalities. See the Senior Citizen Freedom to Work Act of 2000.
- Advanced students could be asked to include an analysis of the fraction of pre-retirement income replaced by social security with different ages for starting to collect benefits. This will obviously be low for higher level of incomes, but the PIA is a large fraction of the AIME for low income levels.
- Analysis of individual decisions can be linked into questions about the Social Security System—its financial stability and possible changes to improve that.

**Summary and Conclusions**

We believe that this case can be presented appropriately to undergraduate students who are personally a long ways from retirement. It has the enormous advantage of not depending on the instructor or a case writer to assemble the information—thus students must do much of the searching and problem refinement that is one of the principal differences between textbook problems and the real world.

This paper has outlined one solution that we believe is within the capabilities of students in engineering economy courses. It may be better approached by teams rather than individuals, and there are easier introductions to case analysis. We have also commented both on ways to break down the case to increase the ratio of learning to effort and on possible extensions to more difficult situations.
This paper has not included any time estimates for different choices of case definition, as our initial assignment of this case to students will occur between submission of this final paper and presentation at the conference. Our hope is that the conference will spark use by others so that we can report the results of that use to instructors, and so that this can be improved to maximize its value to the teaching of engineering economy.

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References